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The Green function of neutral gluons in color magnetic background field at finite temperature

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Abstract

In $SU(2)$ gluodynamics, the tensor structure of the exact Green function of gluons neutral with respect to a homogeneous chromomagnetic field at finite temperature is derived. It is expressed through 10 tensors and corresponding form factors. These tensors constitute an algebra with respect to anticommutation. The structure constants of the algebra are calculated. The spectrum of gluons is derived from the location of poles of this Green function for the case of form factors computed in one-loop order. The high temperature asymptotics for the form factors and the spectra of different gluon states in this limit are calculated.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

Recent investigations of QCD at high temperature revealed an important role of colored magnetic fields. In particular, it has been elucidated in gluodynamics that color magnetic fields are spontaneously created at high temperature [1–3]. It is also reasonable to suppose that the same mechanism, the spontaneous generation of magnetic fields, is also responsible for producing seed magnetic fields in the early universe. From the analysis of the lattice simulations [6, 7], and using the perturbative daisy resummations in the external field at high temperature [3, 8] it was discovered that Abelian chromomagnetic fields of order $gB \sim g^4T^2$, where g is a gauge coupling constant, B is field strength, T is the temperature, are spontaneously created.

To describe the properties of matter in this case one has to calculate the spectra of quarks and gluons in the background field at finite temperature. In the present paper, we construct the exact neutral gluon Green function in the external Abelian chromomagnetic field at finite temperature and investigate its properties. The gluon spectrum is derived from the location of poles of this function. For that we first determine the tensor structure of the neutral gluon Green function in this environment. It is presented as the set of operators and corresponding form factors. These form factors are expressed in terms of the polarization tensor which we have calculated early in one-loop order [5]. Hence the spectra of gluons in the given approximation are calculated. Some features of these states are investigated.

2. The structure of the polarization tensor

We consider $SU(2)$ gluodynamics as the particular case. We divide the gauge field potential $A^a_\mu(x)$ into the background Abelian homogeneous magnetic field $B^a_\mu(x)$ and the quantum fluctuations $Q^a_\mu(x)$,

$$A^a_\mu(x) = B^a_\mu(x) + Q^a_\mu(x). \tag{1}$$

The background field $B^a_\mu(x)$ is directed along the third axis in both color and configuration spaces. Its vector potential is

$$B^a_\mu(x) = \delta^{a3}\delta_{\mu 1}x_2 B. \tag{2}$$

In the field presence it is convenient to turn to the so-called 'charged basis' $W^\pm_\mu = \left(Q^1_\mu \pm \mathrm{i}Q^2_\mu\right)/\sqrt{2}$, $Q_\mu = Q^3_\mu$, with the interpretation of $W^\pm_\mu$ as color charged fields ('charged' gluons) and Q_μ as color neutral fields ('neutral' gluons). The neutral gluon has continuous momentum, whereas the charged one has the discrete Landau levels in perpendicular with respect to the field direction. In [4] the one-loop gluon polarization tensor at zero temperature was derived. In [5] that has been done for the finite temperature case.

In what follows, we use the Feynman gauge where the propagator of neutral gluon in Euclidean's metric with a momentum k_μ is

$$D^{(0)}_{\mu\nu} = \frac{\delta_{\mu\nu}}{k^2}. \tag{3}$$

In the tree approximation, the spectrum can be determined from the location of poles of $D^{(0)}_{\mu\nu}$, that is from the equation $k^2 = 0$.

The exact Green function $D_{\mu\nu}$ of neutral gluons in the field $B^a_\mu(x)$ is a function of two vectors formed from momentum components $h_\lambda = (k_1, k_2, 0, 0)$, $l_\lambda = (0, 0, k_3, k_4)$ and the field induction B. It is given by the Schwinger–Dyson equation which in operator form reads

$$D = \frac{1}{(k^2 - \Pi)}, \tag{4}$$

where Π is the polarization tensor (PT).

As it was shown in [4, 9], in a magnetic field the PT is not transversal. This means that the condition $k_\mu \Pi_{\mu\nu} = 0$ does not hold. A weaker condition,

$$k_\mu \Pi_{\mu\nu} k_\nu = 0. \tag{5}$$

following from the Slavnov–Taylor identity, only holds.

In [5] the following tensor structure of the neutral gluon PT at finite temperature was derived

$$\Pi_{\mu\nu} = \sum_{i=1}^{10} \Pi^{(i)} T^{(i)}_{\mu\nu} \tag{6}$$

with

$$
\begin{aligned}
T^{(1)}_{\lambda\lambda'} &= l^2\delta^{\parallel}_{\lambda\lambda'} - l_\lambda l_{\lambda'}, & T^{(2)}_{\lambda\lambda'} &= h^2\delta^{\perp}_{\lambda\lambda'} - h_\lambda h_{\lambda'},\\
T^{(3)}_{\lambda\lambda'} &= h^2\delta^{\parallel}_{\lambda\lambda'} + l^2\delta^{\perp}_{\lambda\lambda'} - l_\lambda h_{\lambda'} - h_\lambda l_{\lambda'}, & T^{(4)}_{\lambda\lambda'} &= h^2\delta^{\parallel}_{\lambda\lambda'} - l^2\delta^{\perp}_{\lambda\lambda'},\\
T^{(5)}_{\lambda\lambda'} &= i(l_\lambda d_{\lambda'} - d_\lambda l_{\lambda'}) + \mathrm{i}l^2 F_{\lambda\lambda'}, & T^{(6)}_{\lambda\lambda'} &= \mathrm{i}F_{\lambda\lambda'},
\end{aligned} \tag{7}
$$

where $\delta^{\parallel} = \mathrm{diag}(0,0,1,1), \delta^{\perp} = \mathrm{diag}(1,1,0,0), d_\lambda = (k_2, -k_1, 0, 0)$ and the nonzero components of $F_{\lambda\lambda'}$ are $F_{01} = -F_{10} = 1$.

The first four tensors $T^{(i)}$ are transversal, $k_\mu T_{\mu\nu} = 0$, whereas the last two obey only equation (5). At finite temperature, we have to take into consideration the additional vector u_μ—the thermostat velocity. Hence four additional tensors are as follows:

$$
\begin{aligned}
T^{(7)}_{\lambda\lambda'} &= (uk)(u_\lambda l_{\lambda'} + l_\lambda u_{\lambda'}) - \delta^{\parallel}_{\lambda\lambda'}(uk)^2 - u_\lambda u_{\lambda'} l^2,\\
T^{(8)}_{\lambda\lambda'} &= (uk)(u_\lambda h_{\lambda'} + h_\lambda u_{\lambda'}) - \delta^{\perp}_{\lambda\lambda'}(uk)^2 - u_\lambda u_{\lambda'} h^2,\\
T^{(9)}_{\lambda\lambda'} &= \mathrm{i}(u_\lambda d_{\lambda'} - d_\lambda u_{\lambda'}) + \mathrm{i}F_{\lambda\lambda'}(uk),\\
T^{(10)}_{\lambda\lambda'} &= k^2\delta_{\lambda\lambda'} - \frac{(k^2)^2 u_\lambda u_{\lambda'}}{(uk)^2}.
\end{aligned} \tag{8}
$$

In the reference frame of thermostat $u_\mu = (0,0,0,1)$ the scalar product $(uk) = k_4$ is the fourth component of the momentum. The tensors $T^{(7)}$, $T^{(8)}$ and $T^{(9)}$ are transversal and $T^{(10)}$ satisfies the weaker condition (5).

Since $\Pi_{\mu\nu}$ is real and symmetric in its indices, the form factors $\Pi^{(5)}$, $\Pi^{(6)}$ and $\Pi^{(9)}$ equal to zero. It is possible to check that the set of tensors (7)–(8) together with the identity matrix $T^{(0)}_{\mu\nu} = k^2\left(\delta^{\parallel}_{\mu\nu} + \delta^{\perp}_{\mu\nu}\right)$ forms an algebra

$$
\{T^{(i)}, T^{(j)}\} = 2C^{ij}_k T^{(k)}. \tag{9}
$$

Its structure constants C^{ij}_k were calculated from the explicit expressions for the tensors $T^{(i)}$, where the indices run the values $i, j = 0, 1, \ldots, 10$. That is assumed in what follows. Due to completeness of the set of operators $T^{(i)}$, one can obtain D as a linear combination

$$
D_{\mu\nu} = \sum_{i=0}^{10} D^{(i)} T^{(i)}_{\mu\nu}, \tag{10}
$$

where $D^{(i)}$ are some scalar functions of k^2 and l^2 entering the form factors $\Pi^{(i)}$. They will be calculated in the following section.

3. The gluon Green function

First we note that $T^{(i)}$ are functions of $h_\mu = (k_1, k_2, 0, 0), l_\mu = (0, 0, k_3, k_4)$ and $u_\mu = (0,0,0,1)$. The convolution of $T^{(i)}$ and some linear combination of h_μ, l_μ and u_μ is again a linear combination of these vectors with other coefficients, $(\alpha l_\mu + \beta h_\mu + \gamma u_\mu)T^{(i)}_{\mu\nu} = xl_\nu + yh_\nu + zu_\nu$. Let us consider a tensor

$$
P(\alpha,\beta,\gamma,x,y,z)_{\mu\nu} \equiv (\alpha l_\mu + \beta h_\nu + \gamma u_\mu)(xl_\nu + yh_\nu + zu_\mu) \tag{11}
$$

and its convolution with D. From equation (10) we obtain

$$
\begin{aligned}
P(\alpha,\beta,\gamma,x,y,z)_{\mu\nu} D_{\nu\mu} &= (\alpha l_\mu + \beta h_\mu + \gamma u_\mu) D_{\mu\nu}(xl_\nu + yh_\nu + zu_\nu)\\
&= \sum_{i=0}^{10} D^{(i)}(\alpha l_\mu + \beta h_\mu + \gamma u_\mu) T^{(i)}_{\mu\nu}(xl_\nu + yh_\nu + zu_\nu).
\end{aligned} \tag{12}
$$

On the other hand, we can substitute $(k^2-\Pi)^{-1}_{\mu\nu}$ for $D_{\mu\nu}$ in equation (12) and get some functions which depend on the form factors $\Pi^{(i)}$,

$$P(\alpha,\beta,\gamma,x,y,z)_{\mu\nu}\left[\frac{1}{(k^2-\Pi)}\right]_{\nu\mu} = \frac{1}{k^2}\sum_{r=0}^{\infty}(-1)^r\frac{1}{k^{2r}}\left[(\alpha l_\mu+\beta h_\mu+\gamma u_\mu)^T\Pi^r_{\mu\nu}\right](xl_\nu+yh_\nu+zu_\nu). \quad (13)$$

Here we expressed the function of Π in the form of series to find

$$(\alpha l_\mu+\beta h_\mu+\gamma u_\mu)\Pi_{\mu\nu}=(\alpha' l_\nu+\beta' h_\nu+\gamma' u_\mu). \quad (14)$$

In the operator form we get

$$A\begin{pmatrix}\alpha\\ \beta\\ \gamma\end{pmatrix}=\begin{pmatrix}\alpha'\\ \beta'\\ \gamma'\end{pmatrix}, \quad (15)$$

where A is a transformation matrix. Obviously that

$$(\alpha l_\mu+\beta h_\mu+\gamma u_\mu)\Pi^r_{\mu\nu}=A^r(\alpha' l_\nu+\beta' h_\nu+\gamma' u_\mu). \quad (16)$$

So, if we have a function of Π we can replace it by A

$$P(\alpha,\beta,\gamma,x,y,z)_{\mu\nu}\left[\frac{1}{(k^2-\Pi)}\right]_{\nu\mu}=P(\alpha,\beta,\gamma,x,y,z)_{\mu\nu}\left[\frac{1}{(k^2-A)}\right]_{\nu\mu}. \quad (17)$$

In our case, the matrix A has the following elements:

$$\begin{aligned}
&A_{11}=h^2(\Pi_{(3)}+\Pi_{(5)})+(uk)^2(\Pi_{(7)}-\Pi_{(8)});\\
&A_{12}=-l^2\Pi_{(3)}+(uk)^2\Pi_{(8)}; \qquad A_{13}=-h^2\Pi_{(8)};\\
&A_{21}=-h^2\Pi_{(3)}; \qquad A_{22}=l^2(\Pi_{(3)}-\Pi_{(5)}); \qquad A_{23}=h^2\Pi_{(8)};\\
&A_{31}=(uk)^2\Pi_{(1)}; \qquad A_{32}=(uk)^2(\Pi_{(3)}+\Pi_{(8)});\\
&A_{33}=l^2\Pi_{(1)}+h^2(\Pi_{(3)}+\Pi_{(5)})+((uk)^2-h^2)\Pi_{(8)}.
\end{aligned} \quad (18)$$

By specifying the values of coefficients $\alpha,\beta,\gamma,x,y,z$ we can derive the factors $D^{(i)}$ in terms of form factors $(\Pi_{(i)}=\Pi_i)$

$$\begin{aligned}
D^{(0)}&=\frac{B_{11}+B_{12}+B_{21}+B_{22}}{k^2\psi},\\
D^{(1)}&=\frac{\omega+(uk)^2\Pi_7\delta}{k^2-l^2\Pi_1-h^2(\Pi_3+\Pi_5)+(uk)^2\Pi_7},\\
D^{(2)}&=\frac{1}{\psi}\frac{k^2\Pi^2+h^2\Pi_2(\Pi_3+\Pi_5)+h^2\Pi_3^2+(uk)^2\Pi_8(\Pi_2-\Pi_8)}{k^2+h^2\Pi_2+l^2(\Pi_3-\Pi_5)+(uk)^2\Pi_8},\\
D^{(3)}&=-\frac{B_{12}+B_{32}}{\psi h^2[l^2+(uk)^2]},\\
D^{(5)}&=\frac{(uk)^2-k^2}{\psi k^2h^2[(uk)^2-l^2]}\left[B_{11}+B_{13}+B_{31}+\frac{h^2}{(uk)^2-k^2}B_{32}\right],\\
D^{(7)}&=\frac{\omega+(uk)^2\Pi_7\delta}{k^2-l^2\Pi_1-h^2(\Pi_3+\Pi_5)+(uk)^2\Pi_7}-\delta,\\
D^{(8)}&=\frac{1}{\psi h^2[l^2+(uk)^2]}\left[B_{12}+\frac{l^2}{(uk)^2}B_{32}\right],\\
D^{(10)}&=\frac{B_{21}+B_{31}+B_{22}+B_{32}l^2/(uk)^2}{\psi k^4[1-l^2/(uk)^2]},
\end{aligned} \quad (19)$$

where we introduced the notation $\psi = \det[k^2 - A]$, B_{ij} are the matrix elements of $B = (k^2 - A)^{-1}$, $\delta = (B_{31}+B_{32})[l^2-(uk)^2]^{-1}\psi^{-1}$ and $\omega = [k^2\Pi_1 D_0 + h^2\Pi_1(D_3+D_5)+h^2\Pi_3 D_1]$.

By means of equation (19) we expressed the form factors of the Green function in terms of form factors of the PT. The latter are still arbitrary. In the following section we calculate the one-loop approximation to them.

4. Form factors in one-loop order

In [5] the form factors $\Pi^{(i)}(k)$ have been represented as two-parametric integrals and a sum,

$$\Pi^{(i)}(k) = \sum_{N=-\infty}^{\infty} \int_0^\infty \mathrm{d}s\,\mathrm{d}t\, M^{(i)}(s,t)\Theta_T. \tag{20}$$

The explicit form of the functions $M^{(i)}(s,t)$ is

$$\begin{aligned}
M_1 &= 4 - 2\left(\frac{\xi}{q}\right)^2 \cosh(2q),\\
M_2 &= 4\frac{1-\cosh(q)\cosh(\xi)}{(\sinh(q))^2} - 2 + 8\cosh(q)\cosh(\xi),\\
M_3 &= -2\cosh(2q)\frac{\xi\sinh(\xi)}{q\sinh(q)} - 2 + 6\cosh(\xi)\cosh(q),\\
M_4 &= -2 + 2\cosh(q)\cosh(\xi),\\
M_5 &= 2\frac{\xi}{q}\left(\sinh(2q) - \frac{\cosh(q)-\cosh(\xi)}{\sinh(q)}\right) - 6\cosh(q)\sinh(\xi),\\
M_6^{(1)} &= 2\left[\frac{\xi}{q}\coth(q)(1-3(\sinh(q))^2) + \sinh(\xi)\cosh(q)\right]l^2\\
&\quad + 2\left[\frac{\sinh(\xi)}{\sinh(q)}\coth(q)(1-3(\sinh(q))^2) + 2\sinh(\xi)\cosh(q)\right]h^2,\\
M_6^{(2)} &= \frac{\mathrm{i}N}{qT}k_4 2\left(\sinh(2q) - \coth(q)\right),\\
M_7 &= \frac{\mathrm{i}N}{qT}\frac{1}{k_4}\frac{\xi}{q}(-2\cosh(2q)),\\
M_8 &= \frac{\mathrm{i}N}{qT}\frac{1}{k_4}\left(-2\frac{\sinh(\xi)}{\sinh(q)} - 4\sinh(q)\sinh(\xi)\right),\\
M_9 &= \frac{\mathrm{i}N}{qT}2\left[\frac{\cosh(q)-\cosh(\xi)}{\sinh(q)} - \sinh(2q) - 2\sinh(q)\cosh(\xi)\right],\\
M_{10} &= 0,
\end{aligned} \tag{21}$$

with the notations $\xi = s - t, q = s + t$. The calculation was done in the imaginary time formalism, i.e., with a discrete fourth component $p_4 = 2\pi l T$ $(l = 0, 1, 2, \ldots)$ in the loop momentum whereby the external momentum component k_4 was kept arbitrary.

The symmetric under $\xi \to -\xi$ form factors (these are $M_1, \ldots, M_4, M_6^{(2)}, M_9$) go with

$$\Theta_T^s = \Theta(s,t)\tfrac{1}{2}\left(\mathrm{e}^{\frac{\mathrm{i}k_4 N}{qT}t} + \mathrm{e}^{\frac{\mathrm{i}k_4 N}{qT}s}\right)\mathrm{e}^{-\frac{N^2 B}{4T^2 q}}, \tag{22}$$

Table 1. The coefficients a, b, c.

n	a_n	b_n	c_n
1	10.56832 − 0.59082 i	1.85028 + 0.08862 i	1.64935 + 0.29541 i
2	−5.79894 − 7.08982 i	−4.16625 + 3.54491 i	−4.63238 − 1.77245 i
3	1.04427 − 8.86227 i	−4.16625 + 3.54491 i	−2.84292 − 3.10179 i
4	0	0	0
5	−4.21405 − 1.77245 i	−1.60873 + 0.88622 i	−1.58031 + 0.44311 i
6	0	0	0
7	−1.40468 − 0.59082 i	−0.10712 + 0.08862 i	0.13310 + 0.29541 i
8	1.71341 − 3.54491 i	−1.90805 − 1.77245 i	0.38174 − 1.77245 i
9	0	0	0

and the antisymmetric ones (these are $M_5, M_6^{(1)}, M_7, M_8$) go with

$$\Theta_T^a = \Theta(s,t)\frac{1}{2}\left(\mathrm{e}^{\frac{\mathrm{i}k_4 N}{qT}t} - \mathrm{e}^{\frac{\mathrm{i}k_4 N}{qT}s}\right)\mathrm{e}^{-\frac{N^2 B}{4T^2 q}}. \tag{23}$$

There the function $\theta(s,t)$ is

$$\theta(s,t) = \frac{\exp\left(-\frac{k}{B}\left(\delta^{\|}\frac{st}{s+t} + \delta^{\perp}\frac{ST}{S+T}\right)k\right)}{(4\pi)^2(s+t)\sinh(s+t)}, \tag{24}$$

with the abbreviations $S \equiv \tanh s$ and $T \equiv \tanh t$. In these formulae we have put the magnetic field $B = 1$. It can be restored by the substitution $s \to Bs, t \to Bt, T \to T/B$ in equation (21).

In the following, we focus on the spectrum at high temperature $\sqrt{B}/T \ll 0$ in the limit of $k_4 = 0, \vec{k} \to 0$. For this case from the above formulae we calculated the following asymptotic expressions for the form factors,

$$\Pi^{(n)}(k) = \frac{T}{\sqrt{B}(4\pi)^{3/2}}\left(a_n - \frac{l^2}{B}b_n - \frac{h^2}{B}c_n\right) - \theta_n. \tag{25}$$

The corresponding coefficients a, b, c for each form factor are given in table 1. For the function θ_n we get

$$\theta_n = \frac{10}{3}\frac{1}{(4\pi)^2}\ln\left(\frac{T^2}{B}\right), \qquad n = 1, 2, 3; \tag{26}$$

$$\theta_n = 0, \qquad n \neq 1, 2, 3. \tag{27}$$

It is interesting that all these form factors are expressed in terms of Rieman's Zeta-function.

The imaginary part results from the instability of the tachyonic state. This is because the spectrum of charged gluons in a constant magnetic field

$$E_n^2 = p_3^2 + B(2n+1), \qquad n = -1, 0, 1, \ldots, \tag{28}$$

contains a tachyonic mode at $n = -1$. p_3 is momentum along the field direction $B = B_3$ (see for details, for instance, [4]). This state is a peculiarity of non-Abelian gauge fields.

The real part is responsible for the screening of transversal gluon fields. It is important to note that at finite temperature the ratio of the imaginary and the real parts, $\rho = |\mathrm{Im}\,\Pi|/|\mathrm{Re}\,\Pi|$, is an important parameter characterizing the stability of a state. If $\rho < 1$ the state is a quasi-stable one and the state is unstable for $\rho > 1$. In case of small ρ the form factor does not need to be resummed. For $\rho > 1$ the form factor should be resummed and the one-loop results are not reliable.

5. The spectra of gluons in the background field at high temperature

The spectrum of the gluon excitations is determined by the location of poles of the Green function whereby one has to return from the Euclidean representation which was used for the calculations to the Minkowskian one by means of $k_4 \to \mathrm{i}(\omega + \mathrm{i}\epsilon)$. Thereby the momentum component k_4 must be an arbitrary parameter and, if necessary, one has to make the analytic continuation from the discrete values it has in the Matsubara approach, for details see [10]. In our case, k_4 was kept arbitrary in the whole course of calculation of the form factors which resulted in equations (21)–(24).

Now, we derive the spectral equations. That can be done from equation (19) presenting the location of poles of the Green function. There are three spectral equations. Two of them are linear with respect to k^2 and one is cubic in k^2

$$k^2 - h^2\Pi^{(2)} - l^2(\Pi^{(3)} - \Pi^{(5)}) + (uk)^2\Pi^{(8)} = 0, \tag{29}$$

$$k^2 - l^2\Pi^{(1)} - h^2(\Pi^{(3)} + \Pi^{(5)}) + (uk)^2\Pi^{(7)} = 0, \tag{30}$$

$$\psi = 0. \tag{31}$$

The next step is to substitute the form factors $\Pi^{(i)}$ and determine the spectra in the chosen one-loop approximation.

Different tensor structures contribute to different gluon polarization states. To investigate the propagation of a state with a specified polarization one has to calculate the mean value of the Green function in this state. As a particular case of possible modes, let us consider motion along the magnetic field. We put $h = 0$ and consider the high temperature limits for the form factors, substituted into dispersion equations (29), (30).

The results for the transverse state $\langle s = 2|D(\omega, k_3^2)|s = 2\rangle$ at the temperature $T/\sqrt{B} = 20$ is depicted in the plot (figure 1). Here $s = 1, 2$ describe the transversal polarizations of gluons in the background field, as is described in detail in [4, 5] and ω is the frequency, $k_4 \to \mathrm{i}(\omega + \mathrm{i}\epsilon)$.

One mode is stable. It propagates with velocities less than the speed of light. The vacuum polarization acts as a medium for neutral gluons. The other one is tachyonic.

As concerns the modes moving in perpendicular to the field directions, the corresponding form factors $\Pi^{(2)}$ and $\Pi^{(3)}$ have large imaginary parts and need a different treatment which is not discussed here.

6. Conclusion

In $SU(2)$ gluodynamics, we derived the tensor structure of the exact neutral gluon Green function in an Abelian homogeneous magnetic field at finite temperature. It is presented as the linear combination of 10 tensors $T^{(i)}$. It was proved that these tensors form an algebra with respect to the operation of anticommutation, whose structure constants have been calculated. The spectrum of gluons was obtained from the location of poles of these Green function with the one-loop form factors inserted. The equations for the spectrum of the neutral gluons were derived.

It was shown that for the transversal modes moving along the direction of the field the speed of propagation is smaller than the speed of light and that there are new modes created due to the environment. In the one-loop approximation, all the form factors contain imaginary parts because of the well-known tachyonic instability. Our results are aimed as a step toward a resummation of perturbation series expecting a stabilization of the spectrum.

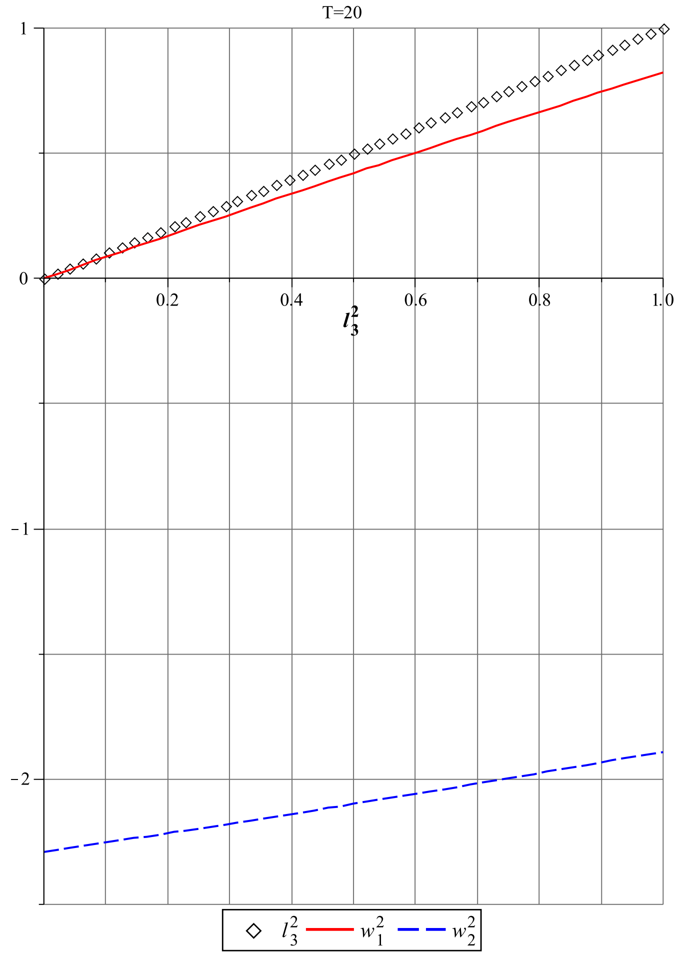

Figure 1. Dispersion relations for the transversal modes in case of motion along the field for $h^2 = 0$ and $T/\sqrt{B} = 20$. The curves represent the dependence of the square of the gluon frequency ω^2 on the square of the momentum, $\vec{k}^2 = k_3^2$. The dotted line is the tree level spectrum $\omega^2 = k_3^2$. The solid line is the first solution of equation (30) and the dashed line is the second solution of this equation.

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